On the Variations of Level and Azimuth of the Transit Circle at the Royal Observatory, Greenwich. By H. H. Turner, M.A., B.Sc.

In Vol. XXIX. of the *Memoirs* of the Royal Astronomical Society, Mr. William Ellis has given a very complete account of the changes of level and azimuth errors of the Transit Circle during the early years of its history (1851–1858). He found that there were fluctuations in the position of the instrument in both these elements which followed very closely the changes of external temperature, especially those of long period, such as the annual variation. As the changes of position of an instrument which has been in constant use for so long a period are of interest, the following tables have been recently formed, showing the simple arithmetical mean of the adopted level errors in each month, and of the adopted azimuth errors in each month, for the period 1851–1884. From the means for each month that for the year has been formed (in the column on the right); and similarly the means for the separate months have been formed (in the bottom line), excluding those years in which the changes mentioned in the notes have been made.

Mean Values of Adopted Level Error of the Transit Circle for each month and for each year since its erection.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	"	"	"	"	"	"	"	"	"	"	"	"	"
1851	− 2·64	− 3·38	− 4·01	− 4·53	− 4·83	− 5·91	− 4·56	− 4·31	− 2·71	− 1·75	− 0·61	− 1·49	− 3·39
1852	− 2·20	− 2·72	− 2·79	− 3·14	− 3·99	− 4·61	− 6·12	− 2·64	− 0·52	− 0·43	− 1·29	− 1·23	− 2·64
1853	− 1·37	− 1·07	− 1·75	− 2·56	− 3·51	− 3·38	− 2·92	− 1·60	− 0·30	0·00	+ 0·63	+ 0·86	− 1·41
1854	− 1·38	− 1·44	− 1·72	− 2·42	− 1·79	− 2·50	− 2·83	− 1·46	− 0·31	+ 1·75	+ 2·59	+ 2·06	− 0·79
1855	+ 1·43	+ 1·70	+ 0·95	− 0·02	− 0·72	− 0·81	− 0·85	+ 0·09	+ 1·94	+ 2·93	+ 3·52	+ 2·47	+ 1·05
1856	+ 2·22	+ 1·78	+ 1·77	+ 0·12	− 0·10	− 0·73	− 0·70	+ 0·29	+ 2·25	+ 3·13	+ 3·78	+ 3·15	+ 1·41
1857	+ 2·70	+ 2·07	+ 1·61	+ 0·82	− 0·13	− 0·92	+ 0·59	+ 1·77	+ 3·49	+ 4·84	+ 5·59	+ 4·66	+ 2·26
1858	+ 4·18	+ 4·16	+ 2·73	+ 1·70	+ 1·50	+ 0·02	+ 2·32	+ 3·43	+ 4·29	+ 5·61	+ 5·62	+ 4·96	+ 3·38
1859	+ 5·00	+ 4·20	+ 3·57	+ 3·49	+ 3·17	+ 1·58	+ 1·89	+ 4·71	+ 6·63	+ 7·39	+ 7·40	+ 6·95	+ 4·66
1860	+ 6·37	+ 5·81	+ 5·08	+ 4·42	+ 2·86	+ 3·44	+ 3·37	+ 4·22	+ 5·42	+ 5·85	+ 6·79	+ 6·33	+ 5·00
1861	+ 5·87	+ 4·94	+ 4·60	+ 5·58	+ 3·71	+ 3·16	+ 4·70	+ 5·11	+ 7·03	+ 7·23	+ 8·44	+ 7·63	+ 5·67
1862	+ 6·93	+ 6·48	+ 5·11	+ 4·79	+ 3·20	+ 3·72	+ 3·99	+ 4·89	+ 6·02	+ 7·16	+ 8·01	+ 7·34	+ 5·64
1863	+ 6·87	+ 7·17	+ 6·38	+ 5·65	+ 4·86	+ 4·87	+ 5·98	+ 6·18	+ 8·45	+ 8·29	+ 8·84	+ 8·68	+ 6·85
1864	+ 8·29	+ 8·09	+ 7·39	+ 4·76	+ 5·66	+ 6·17	+ 6·36	+ 7·66	+ 9·08	+ 9·61	+ 10·05	+ 10·07	+ 7·77
1865	+ 9·79	+ 8·90	+ 8·61	+ 6·33	+ 6·16	+ 6·67	+ 7·09	+ 9·26	+ 9·10	+ 10·97	+ 11·30	+ 10·95	+ 8·76
1866	+ 10·59	+ 9·88	+ 9·72	+ 8·32	+ 8·67	+ 7·35	+ 8·42	+ 9·25	+ 10·65	+ 10·79	+ 10·92	+ 10·98	+ 9·63
1867	+ 10·43	+ 9·63	+ 9·91	+ 7·90	+ 6·94	+ 7·05	+ 7·84	+ 7·32	+ 9·43	+ 12·79	+ 10·72	+ 10·36	+ 9·19

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
1868	+ 9.41″	+ 8.57″	+ 8.31″	+7.46″	+6.26″	+6.37″	+7.06″	+10.15″	+10.78″	+12.44″	+12.61″	+11.51″	+ 9.24″
1869	+11.57	+10.23	+10.74	+8.02	+9.02	+8.28	+7.31	+ 9.45	+ 9.98	+11.89	+11.90	+11.82	+10.02
1870(*a*)	+10.91	+10.69	+ 9.60	+8.49	+7.82	+7.61	+7.94	+10.41	+10.88	+11.39	...	+ 2.77	...
1871	+ 2.30	+ 0.49	+ 0.08	−0.07	−0.70	−0.29	−1.34	− 1.01	− 1.65	+ 2.95	+ 2.99	+ 2.27	+ 1.11
1872	+ 1.84	+ 0.98	+ 0.88	−0.49	−0.68	−2.31	−1.67	+ 0.65	+ 1.53	+ 2.88	+ 1.83	+ 2.40	+ 0.65
1873	+ 1.39	+ 1.13	+ 0.09	−0.17	−1.20	−2.32	−2.45	− 0.63	+ 1.38	+ 2.44	+ 1.82	+ 1.72	+ 0.27
1874	+ 1.15	+ 1.19	− 0.24	−1.74	−0.33	−1.54	−1.85	− 0.21	+ 0.69	+ 1.73	+ 2.51	+ 2.41	+ 0.31
1875	+ 0.43	+ 0.67	− 0.37	−0.95	−2.53	−2.46	−1.94	− 2.41	− 0.69	+ 0.70	+ 0.96	+ 0.25	− 0.69
1876	− 0.46	− 1.25	− 0.89	−1.57	−2.10	−3.99	−3.95	− 2.29	+ 0.08	+ 0.03	+ 0.68	− 0.07	− 1.31
1877(*b*)	− 1.23	− 1.86	− 2.29	−3.09	−3.53	−5.76	−4.48	− 3.92	− 1.56	− 1.46	− 1.00	(− 1.03)	− 2.60
1878	− 1.85	− 3.08	− 2.40	−3.68	−4.45	−6.29	−5.47	− 3.84	− 2.27	− 1.82	− 1.18	− 1.70	− 3.17
1879	− 2.41	− 3.92	− 4.16	−4.64	−5.05	−5.47	−6.62	− 6.61	− 5.05	− 3.95	− 3.07	− 3.12	− 4.51
1880	− 4.62	− 5.52	− 5.85	−6.33	−7.46	−7.98	−8.42	− 7.76	− 5.76	− 4.08	− 3.76	− 5.04	− 6.05
1881(*c*)	− 4.46	− 5.37	− 6.34	−7.39	−8.49	−8.53	...	− 0.75	+ 0.51	+ 1.81	+ 0.87	+ 0.93	...
1882	+ 0.44	− 0.22	− 2.48	−2.12	−3.22	−3.31	−2.24	− 1.39	+ 0.02	+ 0.85	+ 0.88	+ 0.23	− 1.03
1883	− 0.59	− 1.37	− 1.18	−2.84	−3.20	−3.61	−3.49	− 2.77	− 1.26	− 0.13	− 0.06	− 0.05	− 1.71
1884	− 1.34	− 1.50	− 2.92	−2.46	−3.72	−3.96	−4.42	− 3.97	− 1.93	− 0.66	− 0.61	− 1.22	− 2.39
Mean	+ 2.91	+ 2.34	+ 1.83	+0.96	+0.40	−0.12	+0.16	+ 1.34	+ 2.87	+ 3.92	+ 4.19	+ 3.75	+ 2.05

Mean Values of Adopted Azimuth Error of the Transit Circle for each month and for each year since its erection.

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
1851	−7·13	−8″·29	−7″·28	−6″·72	−6′·12	−3″·19	−1″·61	−0″·72	+0″·02	−1″·21	−3″·97	−5″·62	−4″·32
1852	−6·54	−7·96	−8·45	−7·82	−7·57	−6·00	−2·77	−1·32	−0·35	−1·32	−2·70	−3·40	−4·68
1853	−4·28	−7·68	−9·09	−9·82	−6·63	−4·79	−2·35	−0·84	−0·57	−0·87	−3·46	−6·05	−4·70
1854	−7·89	−9·34	−7·44	−8·16	−7·04	−5·47	−3·57	−2·74	−2·07	−2·39	−3·02	−5·12	−5·35
1855	−7·70	−9·25	−9·45	−7·69	−7·59	−4·59	−2·59	−1·03	−0·68	+0·32	−1·67	−5·09	−4·75
1856	−5·76	−7·28	−7·61	−6·53	−6·21	−2·87	−1·40	+1·05	+0·18	+0·49	−3·21	−3·12	−3·52
1857	−5·34	−6·19	−6·61	−5·42	−3·75	−1·96	+0·31	+2·43	+3·34	+2·39	+0·62	−0·61	−1·73
1858	−3·02	−5·55	−7·08	−6·30	−4·46	−0·90	+0·16	+1·85	+1·96	+0·65	−1·35	−1·54	−2·13
1859	−3·37	−3·97	−4·99	−5·36	−4·32	−1·57	+2·47	+4·03	+3·75	+2·73	−0·18	−1·72	−1·04
1860	−2·78	−5·27	−5·60	−7·12	−4·60	−3·55	−2·53	−0·59	−1·02	−1·54	−2·81	−3·95	−3·45
1861	−6·89	−6·71	−7·19	−6·11	−7·31	−4·16	−1·43	+0·77	+0·65	+0·55	−2·11	−3·65	−3·63
1862	−5·31	−4·21	−5·86	−4·80	−3·44	−2·51	−0·68	+0·26	+0·76	+0·01	−2·60	−3·19	−2·63
1863	−5·22	−4·14	−6·01	−5·45	−5·02	−1·93	+0·95	+1·52	+0·57	+0·43	−1·30	−3·26	−2·41
1864	−6·33	−5·75	−7·01	−5·77	−3·30	−2·33	−0·46	+1·51	+1·85	−0·28	−2·47	−2·93	−2·77
1865	−5·29	−6·45	−7·67	−4·92	−3·66	−0·85	+1·23	+1·73	+4·15	+0·59	−0·92	−1·42	−1·96
1866	−2·27	−4·27	−5·08	−3·41	−3·34	+0·50	+2·46	+2·25	+2·66	+1·13	−1·19	−0·80	−0·95
1867	−3·94	−1·87	−3·35	−2·49	−1·69	+0·73	+0·45	+3·23	+4·06	+0·72	−1·28	−3·84	−0·77
1868	−5·58	−5·00	−5·19	−4·92	−2·71	+0·32	+3·22	+4·21	+3·55	+3·09	+0·30	+0·54	−0·68
1869	−2·56	−2·10	−4·62	−3·47	−2·08	−0·94	+2·57	+3·14	+3·35	+2·50	−0·67	−2·36	−0·60
1870	−3·97	−5·60	−5·93	−4·87	−2·74	+0·66	+2·59	+3·54	+2·50	+1·89	{−0·35 / −1·11} (a)	−3·38	−1·34

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
	″	″	″	″	″	″	″	″	″	″	″	″	″
1871	−5·48	−4·18	−4·09	−3·96	−3·25	−1·20	+1·05	+ 3·41	+ 4·09	+ 2·18	−0·94	−2·96	−1·28
1872	−2·30	−2·41	−2·59	−1·90	−1·96	+0·07	+2·77	+ 4·92	+ 4·66	+ 2·34	+1·24	−0·16	+0·39
1873	−0·82	−3·72	−3·11	−1·61	−1·38	+0·10	+3·44	+ 4·89	+ 3·84	+ 2·68	−0·16	−0·89	+0·27
1874	−0·33	−1·72	−1·01	−0·04	+0·77	+3·31	+5·36	+ 5·36	+ 5·88	+ 4·37	+2·52	−0·56	+1·99
1875	−0·65	−1·08	−1·64	−0·31	+2·08	+4·65	+6·42	+ 8·78	+ 9·33	+ 8·09	+5·27	+2·87	+3·65
1876	+1·95	+1·91	+2·18	+2·97	+3·32	+5·68	+9·19	+10·79	+ 9·45	+ 9·16	+6·76	+6·66	+5·83
1877	+5·77	+4·93	+3·44	+3·67	+4·05	+7·38	+8·92	+11·19	+10·60	+ 9·43	+8·45	−1·72 / +5·97 (b)	+6·35
1878	−2·95	−2·61	−1·86	−2·09	−0·23	+0·06	+3·44	+ 5·30	+ 4·91	+ 4·54	+0·49	−1·97	+0·59
1879	−2·93	−2·96	−1·70	−2·59	−1·14	+1·59	+2·54	+ 4·75	+ 4·83	+ 4·38	+1·40	−1·93	+0·52
1880	−3·09	−1·94	−0·31	−0·23	+1·06	+2·13	+4·47	+ 5·61	+ 7·27	+ 4·85	+2·07	+1·62	+1·96
1881	−1·93	−1·40	−0·83	−1·25	+0·63	+2·83	+5·79	(+ 8·16)(c)	+ 7·80	+ 6·11	+5·82	+4·47	(+3·02)
1882	+3·62	+3·35	+2·96	+3·39	+5·30	+6·15	+8·36	+ 9·59	+ 9·26	+ 8·70	+6·44	+4·68	+5·98
1883	+3·47	+2·91	+0·84	+1·63	+4·38	+6·48	+7·06	+ 9·64	+10·07	+ 9·11	+6·74	+5·93	+5·69
1884	+5·43	+4·56	+4·61	+4·45	+6·00	+7·86	+8·91	+10·99	+11·23	+10·59	+9·29	+8·17	+7·67
	−3·27	−3·84	−4·20	−3·63	−2·45	−0·30	+1·85	+ 3·38	+ 3·58	+ 2·55	+0·23	−0·96	Mean

Notes.

(*a*) 1870, Nov. 19.—Eastern Y raised and sheet of thin paper placed under it. The Y was adjusted in azimuth by means of the collimators. The two numbers given for azimuth are the means for the two portions of the month.

(*b*) 1877, Dec. 4.—Western Y moved slightly to N to reduce azimuth error. The upper number for azimuth is the actual monthly mean. The lower has been formed by differencing from 1876 and 1878, for use in the annual variation.

(*c*) 1881, July 27–29.—Eastern Y raised and sheet of paper removed. Y adjusted in azimuth by reference to collimators.

The last columns exhibit the secular change in the position of the instrument. The lowest lines exhibit, after correction for that part of the secular change which takes place throughout the year, the annual variation. I have thought it of interest to express this variation in harmonics of annual and semi-annual period, giving at the same time the corresponding expression for the temperature of the air, whose connection with these fluctuations Mr. Ellis has already pointed out. The results are as follows—the unit of time being one year, and the months being assumed of equal length:—

Level	$-1''\cdot96 \sin(298^\circ\ 45' + t.\ 360^\circ) - 0''\cdot45 \sin(67^\circ\ 55' + 2t.\ 360^\circ)$.
Azimuth	$+3''\cdot75 \sin(204\quad 7 + t.\ 360\ ) + 0''\cdot44 \sin(341\quad 30 + 2t.\ 360)$.
Temperature	$12^\circ\cdot22 \sin(247\quad 13 + t.\ 360\ ) + 1^\circ\cdot23 \sin(30\quad 42 + 2t.\ 360)$.

I have only given two terms of the harmonic series, although the residuals of the means of so many years show traces of being capable of further analysis. They are as follows:—

	Level.	Azimuth.	Temperature.
	$''$	$''$	$^\circ$
January ...	+0·01	−0·13	0·0
February ...	+0·01	+0·06	+0·3
March	+0·04	−0·07	−0·7
April	−0·13	+0·08	+0·5
May	+0·09	−0·04	−0·2
June	+0·02	+0·05	+0·4
July	−0·04	−0·03	−0·3
August	−0·02	−0·02	−0·2
September ...	+0·03	+0·02	+0·3
October	0·00	+0·15	+0·7
November ...	−0·01	−0·32	−1·3
December ...	−0·05	+0·29	+0·7

If these changes are due to temperature, which we may suppose to be only gradually taken up by the masses of stone and metal, and the ground on which the instrument stands, we should expect the lagging effect behind the temperature which is exhibited by the azimuth. The difference of epoch for the first harmonic is about 43°, or about a month and a half; and for the second 48°.

We may get some idea of the kind of phenomena to be expected by studying the law generally assumed for underground temperature, though we must remember that this law is deduced for a homogeneous infinite body bounded by a plane surface, while we are here dealing with a mass of variable density and shape, and that we do not know more than very

roughly which part of it contributes most to the effect under consideration.

A wave of temperature expressible by the term

$$A \sin (nt + \alpha)$$

reaches the depth x under the form

$$Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha),$$

where k is a constant depending on the conductivity of the soil.

If the variation of azimuth were due to the variation of temperature at a definite depth below the surface of some part of the instrument, we should expect the lagging for the second harmonic to be about $\sqrt{2}$ times that for the first harmonic. We see, however, that, although the former is decidedly the greater, it is not quite so large in proportion as this. Again, the ratio of the coefficient of the second harmonic to that of the first is a little greater than the corresponding ratio for temperature, whereas we should expect it to be a little less.

But it is probable that this case of the temperature at a definite depth is not fairly comparable with that before us; we are more likely to get a fair analogy by considering the summation of a series of small changes at continually varying depths, i.e. by integrating the above expression in some way.

Simple integration with respect to the depth from the surface to a considerable distance within the earth would give

$$\int_0^\infty Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha)dx = \frac{A}{k\sqrt{n}} \sin \left(nt - \frac{\pi}{4} + \alpha\right),$$

which would represent, say, the elevation of the surface owing to expansion below. And in this sort of integrated effect we notice an important difference from the former; viz. the lagging is constant for waves of all period, and is nearly 45°. Without laying too much stress on the point, it is interesting to notice that the actual lagging is 43° and 48° for the two harmonics. The truth thus apparently lies between this hypothesis and the former, as we might expect.

On proceeding to consider the level, however, we find that, so far from its lagging behind the temperature, it actually precedes it by nearly as much as the azimuth falls behind. In considering a periodic term it is of course always possible to express it as either preceding another of like period by, say, an eighth-period, or following it by seven-eighths; or if we are at liberty to consider a crest of one to correspond to either a crest or trough of the other, we may express the same anticipation of one-eighth of a period as a lagging of three-eighths. The only reason for considering a decrease of level to correspond to an increase of temperature is that such is found to be the case in the irregular and sudden changes which occur from day to day;

but it is of course quite possible that these rapid changes may be caused by the expansion of quite a different part of the instrument, e.g. much nearer the surface of some part of it. We have then the alternative of considering that the level lags at least $4\frac{1}{2}$ months behind the temperature, perhaps $10\frac{1}{2}$ months; and when we consider that this would mean that the seat of these changes would be at a depth of about 24 feet (or perhaps 56 feet) if the conductivity of the parts of the instrument be even so small as that of rock, this does not seem very likely.*

I was led therefore to speculate as to whether this apparent anticipation of the temperature might not be real; and I venture to hazard the suggestion that a difference of conductivities in the two piers of the transit circle would produce such an effect. If we differentiate the expression

$$Ae^{-kx\sqrt{n}} \sin (nt - kx\sqrt{n} + \alpha)$$

to k we obtain

$$-Ak\sqrt{n}\, e^{-kx\sqrt{n}} \sin \left(nt - kx\sqrt{n} + \frac{\pi}{4} + \alpha\right);$$

that is, this differential effect anticipates the former by an eighth-period, and the relative value of the terms of short period for which n is larger is increased in the proportion of $\sqrt{n}$ to 1. This result is not to be taken numerically, but only as a very rough qualitative illustration of the way in which the temperature possibly influences the level. It will be noticed that the supposition of a difference of conductivities will thus explain the apparent anticipation by the level of the change of temperature, and at the same time the large relative value of the second harmonic in the case of level, which is much more marked than in the case of azimuth.

Now, we have above differentiated the expression for the temperature at a definite point within the mass; but it is probable that some process of integration should first have been employed as in the case of azimuth. We have however previously studied the general effect of integration, and know that it is in the opposite direction to that just mentioned. Generally we should expect these two causes—the gradual filtering of the temperature into the solid mass producing an effect of retardation, and the difference of conductivity between the various parts producing an anticipatory effect, to counterbalance one another to a considerable extent, both in the case of azimuth and level. They have been assigned respectively to the one and the other, first for simplicity, and secondly because the *amount* of the effect to be expected from these respective causes compared with the actual differences of epoch observed in the two cases seems to point to the fact that the variation of azimuth is mainly due to the first, and of level to the second. If it be worth while to

* See *Greenwich Observations* for 1860. Reduction of the observations of the deep-sunk thermometers, by Prof. Everett.

hazard further conjecture as to the reason of this difference of behaviour of the instrument in the two elements we might suppose more definitely that the level error is caused by a warming of the eastern pier more rapidly than the western, whose conductivity is not so great; so that it expands vertically and affects the level, while at the same time the lateral expansion, being symmetrical about the central vertical plane which passes nearly through the pivots, would not affect the azimuth. The piers, or that part of them at least which supports the pivots, not being very thick, the lagging due to gradual conduction will not be very large, and we have the effect of difference of conductivity nearly free from any counteraction.

On the other hand, we must refer the azimuthal variation to changes of temperature probably at some depth below the surface of the soil, which do not happen to affect the level appreciably.

I have examined the variations of level and azimuth at other observatories for a few years to see if there is any similarity between these fluctuations for different instruments. It is somewhat difficult, without expending more time than I can well spare at present, to disentangle their annual variations from changes, secular and irregular, and generally larger than those noticed at Greenwich. I look forward with interest to a discussion by Mr. Finlay of the errors of the Cape Transit Circle promised in the last volume of "Cape Observations" (1879–81). Roughly speaking, there does not seem to be much similarity between the fluctuations of position of different instruments; for instance, the difference of epoch, which has been chiefly considered above, is very variable. And indeed Mr. Ellis pointed out in his paper above referred to how largely the variations of the present Transit Circle differ from those of Troughton's Transit, which occupied nearly the same site. As I have already said, however, in suggesting the above explanations of the changes in position of the Greenwich Transit Circle, we have at our disposal two causes which, combined in varying relative proportion, are capable of explaining fluctuations following or anticipating the temperature by very different periods of time.

On the Formulæ for Computing the Apparent Positions of a Satellite, and for Correcting the Assumed Elements of its Orbit. By A. Marth.

The methods of computation connected with the investigation of a satellite's orbit are comparatively simple and convenient, if they are duly selected to suit directly the coordinates furnished by the observations. If polar-coordinates have been observed, there exists apparently no good reason for not employing polar-coordinates also in the corresponding computations. The present